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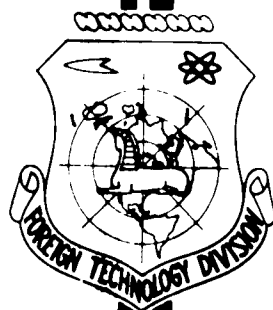
## TRANSLATION

THE TEMPERATURE CALCULATION OF AN ORTHOTROPIC PLATE  
WITH TRANSVERSE SHEAR TAKEN INTO CONSIDERATION

By

S. M. Durgar'yan

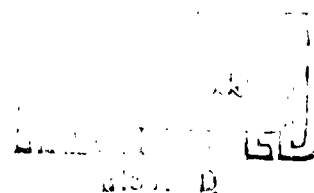
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## UNEDITED ROUGH DRAFT TRANSLATION

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THE TEMPERATURE CALCULATION OF AN ORTHOTROPIC PLATE  
WITH TRANSVERSE SHEAR TAKEN INTO CONSIDERATION

S. M. Durgar'yan  
(Yerevan)

1. Statement of the problem. We will examine a rectangular orthotropic plate of constant thickness  $h$  referred to rectangular cartesian coordinates. We will combine the coordinate plane  $xoy$  with the average plane of an undeformed plate, we will direct the coordinate axes along the principal directions of elasticity of the plate material, and we will set the coordinate origin at one of the apexes of the plate. The deformation factors  $E_1$ ,  $\mu_{1j}$ ,  $G_{1j}$  and the thermal coefficients of expansion  $\beta_1$  will be considered as given functions of temperature  $T$ , changing according to a known law.

Here, as usual,  $E_1$  and  $\beta_1$  are, respectively, Young's modulus and the thermal coefficient of expansion along the coordinate axis  $\underline{1}$ ; the Poisson coefficient  $\mu_{1j}$  characterizes contraction in the direction  $\underline{j}$  during expansion in direction  $\underline{1}$ ; the shear modulus  $G_{1j}$  characterizes the change in angle between principal directions  $\underline{1}$  and  $\underline{j}$ .

We will assume that the relative deformation throughout the plate is conditioned by free thermal expansion

$$\epsilon_z = \beta_z T \quad (1.1)$$

Using as our basis a theory developed by S. A. Ambartsumyan [1], we will consider that the tangential stresses  $\tau_{xz}$  and  $\tau_{yz}$  can be represented as

$$\tau_{xz} = f_1(z), \varphi(x, y), \quad \tau_{yz} = f_2(z) \psi(x, y) \quad (1.2)$$

where  $\varphi(x, y)$  and  $\psi(x, y)$  are unknown functions,  $f_1(z)$  are given functions which characterize the law of change of tangential stresses  $\tau_{xz}$  and  $\tau_{yz}$  throughout the plate;  $f_1(\pm h/2) = 0$ .

2. Derivation of the basic equations. Using the accepted representation of tangential stresses (1.2), having integrated over  $z$  the third differential equation of equilibrium of a volume element (assuming absence of volumetric forces) and having designated

$$F_i(z) = \int_0^z f_i(z) dz \quad (2.1)$$

we will obtain

$$\sigma_z(x, y, z) = \sigma_0(x, y) - F_1(z) \frac{\partial \varphi}{\partial x} - F_2(z) \frac{\partial \psi}{\partial y} \quad (2.2)$$

where  $\sigma_0(x, y)$  is the integration function describing the distribution of normal stresses along the average plane of the plate ( $z = 0$ ) and is determined from the conditions

$$\sigma_z(x, y, \pm h/2) = 0 \quad (2.3)$$

Taking into account (2.3), from (2.2) we obtain the value of the integration function

$$\sigma_0(x, y) = \frac{1}{2} \left[ F_1\left(\frac{h}{2}\right) + F_1\left(-\frac{h}{2}\right) \right] \frac{\partial \varphi}{\partial x} + \frac{1}{2} \left[ F_2\left(\frac{h}{2}\right) + F_2\left(-\frac{h}{2}\right) \right] \frac{\partial \psi}{\partial y}$$

and also the differential dependence

$$\left[F_1\left(\frac{h}{2}\right) - F_1\left(-\frac{h}{2}\right)\right] \frac{\partial \varphi}{\partial x} + \left[F_2\left(\frac{h}{2}\right) - F_2\left(-\frac{h}{2}\right)\right] \frac{\partial \psi}{\partial y} = 0 \quad (2.4)$$

which, as we can easily see, is the third equation of equilibrium of the plate element [1].

Using the found value of the integration function  $\sigma_0(x, y)$  and having set

$$c_i = \frac{1}{2} \left[ F_i\left(\frac{h}{2}\right) + F_i\left(-\frac{h}{2}\right) \right] \quad (i = 1, 2) \quad (2.5)$$

for stress  $\sigma_z(x, y, z)$  we obtain the expression

$$\sigma_z(x, y, z) = [c_1 - F_1(z)] \frac{\partial \varphi}{\partial x} + [c_2 - F_2(z)] \frac{\partial \psi}{\partial y} \quad (2.6)$$

We know from earlier articles ([4], pp. 74-75; [1], pp. 315-316) that it is advantageous to accept the functions  $f_1(z)$  in the form

$$f(z) = f_1(z) = f_2(z) = \frac{1}{2} \left( \frac{h^2}{4} - z^2 \right) \quad (2.7)$$

From (2.1), by virtue of (2.7), we obtain

$$F(z) = F_1(z) = F_2(z) = \frac{z}{2} \left( \frac{h^2}{4} - \frac{z^2}{3} \right)$$

Thus, taking into account (2.4)-(2.6), we will have  $c_1 = c_2 = \sigma_z(x, y, z) = 0$ .

It is easily ascertained that stress  $\sigma_z$  is equal to zero in all cases when the functions  $f_1(z)$  and  $f_2(z)$  are even functions of the coordinate  $z$  and are equal to each other.

From this, in subsequent calculations we will set  $\sigma_z = 0$ .

On the basis of Franz Neuman's hypothesis [2], from the generalized Hooke law we have

$$\begin{aligned} \sigma_x &= B_{11}\epsilon_x + B_{12}\epsilon_y + Q_1 T, & \sigma_y &= B_{12}\epsilon_x + B_{22}\epsilon_y + Q_2 T \\ \tau_{xy} &= B_{44}\epsilon_{xy}, & \tau_{yz} &= B_{44}\epsilon_{yz}, & \tau_{xz} &= B_{44}\epsilon_{xz} \end{aligned} \quad (2.8)$$

where, as usual (for example, [1]),

$$\begin{aligned} B_{11} &= \frac{E_x}{1 - \mu_{xy}\mu_{yx}}, & B_{22} &= \frac{E_y}{1 - \mu_{xy}\mu_{yx}}, & B_{12} &= \frac{E_x\mu_{yx}}{1 - \mu_{xy}\mu_{yx}} = \frac{E_y\mu_{xy}}{1 - \mu_{xy}\mu_{yx}} \\ Q_1 &= -\frac{E_x}{1 - \mu_{xy}\mu_{yx}}(\beta_x + \mu_{yx}\beta_y), & Q_2 &= -\frac{E_y}{1 - \mu_{xy}\mu_{yx}}(\mu_{xy}\beta_x + \beta_y) \\ B_{44} &= G_{yz}, & B_{55} &= G_{xz}, & B_{66} &= G_{xy} \end{aligned} \quad (2.9)$$

Using the known relations between displacements and deformations and also taking into account (1.1), (1.2) and the last two equations of (2.8), for projections of displacement  $u_x$ ,  $u_y$  and  $u_z$  we obtain

$$u_x = u - z \frac{\partial w}{\partial x} - I_x + \varphi \Gamma_1, \quad u_y = v - z \frac{\partial w}{\partial y} - I_y + \psi \Gamma_2, \quad u_z = w + I^* \quad (2.10)$$

Here

$$\begin{aligned} u &= u_x|_{z=0}, & v &= u_y|_{z=0}, & w &= u_z|_{z=0} \\ I^* &= I^*(x, y, z) = \int_0^z \beta_z T dz, & I_1 &= I_1(x, y, z) = \int_0^z \frac{\partial I^*}{\partial x} dz \\ I_2 &= I_2(x, y, z) = \int_0^z \frac{\partial I^*}{\partial y} dz, & I_3 &= I_3(x, y, z) = \int_0^z \frac{\partial I^*}{\partial z} dz \end{aligned} \quad (2.11)$$

According to the known formulas for main stresses

$$\begin{aligned} \sigma_x &= B_{11} \epsilon_x + B_{12} \epsilon_y + B_{13} \epsilon_z + B_{14} \gamma_{xy} + B_{15} \gamma_{yz} + B_{16} \gamma_{xz} \\ \sigma_y &= B_{12} \epsilon_x + B_{22} \epsilon_y + B_{23} \epsilon_z + B_{24} \gamma_{xy} + B_{25} \gamma_{yz} + B_{26} \gamma_{xz} \\ \sigma_z &= B_{13} \epsilon_x + B_{23} \epsilon_y + B_{33} \epsilon_z + B_{34} \gamma_{xy} + B_{35} \gamma_{yz} + B_{36} \gamma_{xz} \\ \tau_{xy} &= B_{14} \epsilon_x + B_{24} \epsilon_y + B_{34} \epsilon_z + B_{44} \gamma_{xy} + B_{45} \gamma_{yz} + B_{46} \gamma_{xz} \\ \tau_{yz} &= B_{15} \epsilon_x + B_{25} \epsilon_y + B_{35} \epsilon_z + B_{45} \gamma_{xy} + B_{55} \gamma_{yz} + B_{56} \gamma_{xz} \\ \tau_{xz} &= B_{16} \epsilon_x + B_{26} \epsilon_y + B_{36} \epsilon_z + B_{46} \gamma_{xy} + B_{56} \gamma_{yz} + B_{66} \gamma_{xz} \end{aligned} \quad (2.12)$$

According to (1) and (2.12) and the known formulas it is not difficult to find the internal stresses and moments by means of the basic

unknowns  $\underline{u}$ ,  $\underline{v}$ ,  $\underline{w}$ ,  $\varphi$ , and  $\psi$

$$\begin{aligned}
 T_x &= \int_{-h/2}^{h/2} \sigma_x dz = C_{11} \frac{\partial u}{\partial x} + C_{12} \frac{\partial v}{\partial y} - K_{11} \frac{\partial^2 w}{\partial x^2} - K_{12} \frac{\partial^2 w}{\partial y^2} + C_{111} \varphi + \\
 &\quad + C_{122} \psi + C_{111} \frac{\partial \varphi}{\partial x} + C_{122} \frac{\partial \psi}{\partial y} - R_{110} x - R_{120} y + \theta_{10} \\
 T_y &= \int_{-h/2}^{h/2} \sigma_y dz = C_{12} \frac{\partial u}{\partial x} + C_{22} \frac{\partial v}{\partial y} - K_{12} \frac{\partial^2 w}{\partial x^2} - K_{22} \frac{\partial^2 w}{\partial y^2} + C_{121} \varphi + \\
 &\quad + C_{222} \psi + C_{121} \frac{\partial \varphi}{\partial x} + C_{222} \frac{\partial \psi}{\partial y} - R_{120} x - R_{220} y + \theta_{20} \\
 S &= \int_{-h/2}^{h/2} \tau_{xy} dz = C_{66} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - 2K_{66} \frac{\partial^2 w}{\partial x \partial y} + C_{661} \varphi + \\
 &\quad + C_{662} \psi + C_{661} \frac{\partial \varphi}{\partial y} + C_{662} \frac{\partial \psi}{\partial x} - H_{y0} x - H_{x0} y \\
 N_x &= \int_{-h/2}^{h/2} \tau_{xz} dz = 2\varphi F \left( \frac{h}{2} \right); \quad N_y = \int_{-h/2}^{h/2} \tau_{yz} dz = 2\psi F \left( \frac{h}{2} \right) \\
 M_x &= \int_{-h/2}^{h/2} \sigma_x z dz = K_{11} \frac{\partial u}{\partial x} + K_{12} \frac{\partial v}{\partial y} - D_{11} \frac{\partial^2 w}{\partial x^2} - D_{12} \frac{\partial^2 w}{\partial y^2} + K_{111} \varphi + \\
 &\quad + K_{122} \psi + K_{111} \frac{\partial \varphi}{\partial x} + K_{122} \frac{\partial \psi}{\partial y} - R_{111} x - R_{121} y + \theta_{11} \\
 M_y &= \int_{-h/2}^{h/2} \sigma_y z dz = K_{12} \frac{\partial u}{\partial x} + K_{22} \frac{\partial v}{\partial y} - D_{12} \frac{\partial^2 w}{\partial x^2} - D_{22} \frac{\partial^2 w}{\partial y^2} + K_{121} \varphi + \\
 &\quad + K_{222} \psi + K_{121} \frac{\partial \varphi}{\partial x} + K_{222} \frac{\partial \psi}{\partial y} - R_{121} x - R_{221} y + \theta_{21} \\
 H &= \int_{-h/2}^{h/2} \tau_{xy} z dz = K_{66} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - 2D_{66} \frac{\partial^2 w}{\partial x \partial y} + K_{661} \varphi + \\
 &\quad + K_{662} \psi + K_{661} \frac{\partial \varphi}{\partial y} + K_{662} \frac{\partial \psi}{\partial x} - H_{y1} x - H_{x1} y
 \end{aligned} \tag{2.13}$$

Here we use the designations

$$\begin{aligned}
 C_i &= \int_{-h/2}^{h/2} B_i dz, \quad K_i = \int_{-h/2}^{h/2} B_i z dz, \quad D_i = \int_{-h/2}^{h/2} B_i z^2 dz, \quad C_{ij} = \int_{-h/2}^{h/2} B_i \Gamma_j dz \\
 K_{ij} &= \int_{-h/2}^{h/2} B_i \Gamma_j z dz, \quad C_{ij}^v = \int_{-h/2}^{h/2} B_i \frac{\partial \Gamma_j}{\partial v} dz, \quad K_{ij}^v = \int_{-h/2}^{h/2} B_i \frac{\partial \Gamma_j}{\partial v} z dz \\
 &\quad (i = 11, 12, 22, 66; j = 1, 2; v = x, y) \\
 H_{ij}^v &= \int_{-h/2}^{h/2} B_{66} \frac{\partial \Gamma_i}{\partial v} z^v dz \quad (i = x, y; j = y, x; v = 0, 1; i \neq j) \\
 R_{ij}^v &= \int_{-h/2}^{h/2} B_i z^j \frac{\partial \Gamma_v}{\partial v} dz \quad (i = 11, 12, 22; v = x, y; j = 0, 1) \\
 \theta_{ij} &= \int_{-h/2}^{h/2} Q_i T_j dz \quad (i = 1, 2; j = 0, 1)
 \end{aligned} \tag{2.14}$$

Introducing (2.13) into the equations of equilibrium of the plate element, for determining the five unknowns  $u$ ,  $v$ ,  $w$ ,  $\varphi$ , and  $\psi$  we obtain the following system of five differential equations with variable coefficients:

$$\begin{aligned} & \left[ \frac{\partial}{\partial x} \left( C_{11} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( C_{12} \frac{\partial}{\partial y} \right) \right] u + \left[ \frac{\partial}{\partial x} \left( C_{12} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( C_{22} \frac{\partial}{\partial y} \right) \right] v - \\ & - \left[ \frac{\partial}{\partial x} \left( K_{11} \frac{\partial^2}{\partial x^2} + K_{12} \frac{\partial^2}{\partial y^2} \right) + 2 \frac{\partial}{\partial y} \left( K_{12} \frac{\partial^2}{\partial x \partial y} \right) \right] w + \\ & + \left[ \frac{\partial}{\partial x} \left( C_{111} \frac{\partial}{\partial x} + C_{111}^* \frac{\partial^2}{\partial x^2} \right) + \frac{\partial}{\partial y} \left( C_{121} \frac{\partial}{\partial y} + C_{121}^* \frac{\partial^2}{\partial y^2} \right) \right] \varphi + \\ & + \left[ \frac{\partial}{\partial x} \left( C_{122} \frac{\partial}{\partial x} + C_{122}^* \frac{\partial^2}{\partial x^2} \right) + \frac{\partial}{\partial y} \left( C_{222} \frac{\partial}{\partial y} + C_{222}^* \frac{\partial^2}{\partial y^2} \right) \right] \psi = \\ & = \frac{\partial (R_{110}^* + R_{120}^* - \theta_{10})}{\partial x} + \frac{\partial (H_{10}^* + H_{20}^*)}{\partial y} \end{aligned} \quad (2.15)$$

$$\begin{aligned} & \left[ \frac{\partial}{\partial x} \left( C_{22} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( C_{12} \frac{\partial}{\partial y} \right) \right] u + \left[ \frac{\partial}{\partial x} \left( C_{12} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( C_{22} \frac{\partial}{\partial y} \right) \right] v - \\ & - \left[ 2 \frac{\partial}{\partial x} \left( K_{12} \frac{\partial^2}{\partial x \partial y} \right) + \frac{\partial}{\partial y} \left( K_{12} \frac{\partial^2}{\partial x^2} + K_{22} \frac{\partial^2}{\partial y^2} \right) \right] w + \\ & + \left[ \frac{\partial}{\partial x} \left( C_{121} \frac{\partial}{\partial x} + C_{121}^* \frac{\partial^2}{\partial x^2} \right) + \frac{\partial}{\partial y} \left( C_{111} \frac{\partial}{\partial y} + C_{111}^* \frac{\partial^2}{\partial y^2} \right) \right] \varphi + \\ & + \left[ \frac{\partial}{\partial x} \left( C_{122} \frac{\partial}{\partial x} + C_{122}^* \frac{\partial^2}{\partial x^2} \right) + \frac{\partial}{\partial y} \left( C_{222} \frac{\partial}{\partial y} + C_{222}^* \frac{\partial^2}{\partial y^2} \right) \right] \psi = \\ & = \frac{\partial (R_{210}^* + R_{220}^* - \theta_{20})}{\partial x} + \frac{\partial (H_{20}^* + H_{10}^*)}{\partial y} \end{aligned}$$

For convenience, the designation  $\frac{\partial^2 \Phi}{\partial x^2} = \frac{\partial^2 \Phi}{\partial y^2} = \Phi$  is used in (2.15)-(2.19); from this,

$$\frac{\partial}{\partial x} \left( \frac{\partial^2}{\partial x^2} \right) = \frac{\partial}{\partial x} \left( \frac{\partial^2}{\partial y^2} \right) = \frac{\partial}{\partial x}, \quad \frac{\partial}{\partial y} \left( \frac{\partial^2}{\partial x^2} \right) = \frac{\partial}{\partial y} \left( \frac{\partial^2}{\partial y^2} \right) = \frac{\partial}{\partial y} \text{ etc.}$$

3. Certain particular cases. The case when the temperature changes only throughout the plate,  $T = T(z)$ . In the examined case the system of equations (2.15)-(2.19) is simplified and reduces to the following system of five uniform differential equations with constant coefficients

$$\begin{aligned} (C_{11} \frac{\partial^2}{\partial x^2} + C_{44} \frac{\partial^2}{\partial y^2}) u + (C_{13} + C_{44}) \frac{\partial^2 v}{\partial x \partial y} - [K_{11} \frac{\partial^2}{\partial x^2} + (K_{12} + 2K_{44}) \frac{\partial^2}{\partial x \partial y}] w + \\ + (C_{111} \frac{\partial^2}{\partial x^2} + C_{441} \frac{\partial^2}{\partial y^2}) \varphi + (C_{112} + C_{442}) \frac{\partial^2 \psi}{\partial x \partial y} = 0 \end{aligned} \quad (3.1)$$

$$\begin{aligned} (C_{13} + C_{44}) \frac{\partial^2 u}{\partial x \partial y} + (C_{44} \frac{\partial^2}{\partial x^2} + C_{22} \frac{\partial^2}{\partial y^2}) v - [(K_{12} + 2K_{44}) \frac{\partial^2}{\partial x^2 \partial y} + K_{22} \frac{\partial^2}{\partial y^2}] w + \\ + (C_{111} + C_{441}) \frac{\partial^2 \varphi}{\partial x \partial y} + (C_{442} \frac{\partial^2}{\partial x^2} + C_{222} \frac{\partial^2}{\partial y^2}) \psi = 0 \end{aligned} \quad (3.2)$$

$$\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} = 0 \quad (3.3)$$

$$\begin{aligned} (K_{11} \frac{\partial^2}{\partial x^2} + K_{44} \frac{\partial^2}{\partial y^2}) u + (K_{12} + K_{44}) \frac{\partial^2 v}{\partial x \partial y} - [D_{11} \frac{\partial^2}{\partial x^2} + (D_{12} + 2D_{44}) \frac{\partial^2}{\partial x \partial y}] w + \\ + [K_{111} \frac{\partial^2}{\partial x^2} + K_{441} \frac{\partial^2}{\partial y^2} - 2F \left( \frac{h}{2} \right)] \varphi + (K_{112} + K_{442}) \frac{\partial^2 \psi}{\partial x \partial y} = 0 \end{aligned} \quad (3.4)$$

$$\begin{aligned} (K_{12} + K_{44}) \frac{\partial^2 u}{\partial x \partial y} + (K_{44} \frac{\partial^2}{\partial x^2} + K_{22} \frac{\partial^2}{\partial y^2}) v - [(D_{12} + 2D_{44}) \frac{\partial^2}{\partial x^2 \partial y} + D_{22} \frac{\partial^2}{\partial y^2}] w + \\ + (K_{111} + K_{441}) \frac{\partial^2 \varphi}{\partial x \partial y} + [K_{442} \frac{\partial^2}{\partial x^2} + K_{222} \frac{\partial^2}{\partial y^2} - 2F \left( \frac{h}{2} \right)] \psi = 0 \end{aligned} \quad (3.5)$$

The case when the temperature changes only along the coordinate axes  $x$  and  $y$ ,  $T = T(x, y)$ . In this case we have

$$\begin{aligned} B_i = B_i(x, y), \quad Q_j = Q_j(x, y) \quad (i = 11, 12, 22, 44, 55, 66; \quad j = 1, 2) \\ I^* = \beta_z T_z, \quad I_x = \frac{1}{2} z^2 \frac{\partial (\beta_z T)}{\partial x}, \quad I_y = \frac{1}{2} z^2 \frac{\partial (\beta_z T)}{\partial y} \end{aligned} \quad (3.6)$$

Using (2.7), from (2.1) and the last two equations of (2.11) we obtain

$$\Gamma_1 = \frac{1}{2B_{44}} \left( \frac{h^3}{4} - \frac{z^3}{3} \right) z, \quad \Gamma_2 = \frac{1}{2B_{44}} \left( \frac{h^3}{4} - \frac{z^3}{3} \right) z, \quad 2F \left( \frac{h}{2} \right) = \frac{h^3}{12} \quad (3.7)$$

Taking into account (3.6) and (3.7), from (2.14) we will have

$$\begin{aligned} C_i &= B_i h, \quad D_i = B_i \frac{h^3}{12}, \quad K_i = C_{i1} = C_{i2} = C_{i1}^v = C_{i2}^v = 0, \quad K_{i1} = \frac{B_i}{B_{44}} \frac{h^4}{120} \\ K_{i2} &= \frac{B_i}{B_{44}} \frac{h^4}{120}, \quad K_{i1}^v = B_i \frac{\partial}{\partial v} \left( \frac{1}{B_{44}} \right) \frac{h^4}{120}, \quad K_{i2}^v = B_i \frac{\partial}{\partial v} \left( \frac{1}{B_{44}} \right) \frac{h^4}{120} \\ &\quad (i = 11, 12, 22, 66; v = x, y) \\ R_{i0}^v &= \frac{B_i h^3}{24} \frac{\partial^2 (\beta_z T)}{\partial v^2}, \quad H_{j0}^v = \frac{B_{44} h^3}{24} \frac{\partial^2 (\beta_z T)}{\partial v \partial j}, \quad R_{i1}^v = H_{j1}^v = 0 \\ &\quad (i = 11, 12, 22; v = x, y; j = y, x) \\ \theta_{i0} &= Q_i T h, \quad \theta_{i1} = 0 \quad (i = 1, 2) \end{aligned} \quad (3.8)$$

Inserting (3.8) into (2.15)-(2.19) we obtain the following system of differential equations with variable coefficients

$$\begin{aligned} \left[ \frac{\partial}{\partial x} \left( B_{11} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( B_{44} \frac{\partial}{\partial y} \right) \right] u + \left[ \frac{\partial}{\partial x} \left( B_{12} \frac{\partial}{\partial y} \right) + \frac{\partial}{\partial y} \left( B_{44} \frac{\partial}{\partial x} \right) \right] v = \\ = \frac{B_{11} h^3}{24} \frac{\partial^3 (\beta_z T)}{\partial x^3} + \frac{(B_{12} + 2B_{44}) h^3 \partial^3 (\beta_z T)}{24 \partial x \partial y^2} - \frac{\partial (Q_1 T)}{\partial x} \end{aligned} \quad (3.9)$$

$$\begin{aligned} \left[ \frac{\partial}{\partial x} \left( B_{44} \frac{\partial}{\partial y} \right) + \frac{\partial}{\partial y} \left( B_{12} \frac{\partial}{\partial x} \right) \right] u + \left[ \frac{\partial}{\partial x} \left( B_{44} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left( B_{22} \frac{\partial}{\partial y} \right) \right] v = \\ = \frac{B_{22} h^3}{24} \frac{\partial^3 (\beta_z T)}{\partial y^3} + \frac{(B_{12} + 2B_{44}) h^3 \partial^3 (\beta_z T)}{24 \partial x^2 \partial y} - \frac{\partial (Q_2 T)}{\partial y} \\ \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y} = 0 \\ - \left[ \frac{\partial}{\partial x} \left( B_{11} \frac{\partial^2}{\partial x^2} + B_{12} \frac{\partial^2}{\partial y^2} \right) + 2 \frac{\partial}{\partial y} \left( B_{44} \frac{\partial^2}{\partial x \partial y} \right) \right] w + \\ + \left\{ \frac{h^3}{10} \frac{\partial}{\partial x} \left[ B_{11} \frac{\partial}{\partial x} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial x^2} \right) \right] + \frac{h^3}{10} \frac{\partial}{\partial y} \left[ B_{44} \frac{\partial}{\partial y} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial y^2} \right) \right] - 1 \right\} \Phi + \\ + \frac{h^3}{10} \left\{ \frac{\partial}{\partial x} \left[ B_{12} \frac{\partial}{\partial y} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial y^2} \right) \right] + \frac{\partial}{\partial y} \left[ B_{44} \frac{\partial}{\partial x} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial x^2} \right) \right] \right\} \Psi = 0 \\ - \left[ 2 \frac{\partial}{\partial x} \left( B_{44} \frac{\partial^2}{\partial x \partial y} \right) + \frac{\partial}{\partial y} \left( B_{12} \frac{\partial^2}{\partial x^2} + B_{22} \frac{\partial^2}{\partial y^2} \right) \right] w + \\ + \frac{h^3}{10} \left\{ \frac{\partial}{\partial x} \left[ B_{44} \frac{\partial}{\partial y} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial y^2} \right) \right] + \frac{\partial}{\partial y} \left[ B_{12} \frac{\partial}{\partial x} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial x^2} \right) \right] \right\} \Phi + \\ + \left\{ \frac{h^3}{10} \cdot \frac{\partial}{\partial x} \left[ B_{44} \frac{\partial}{\partial x} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial x^2} \right) \right] + \frac{h^3}{10} \frac{\partial}{\partial y} \left[ B_{22} \frac{\partial}{\partial y} \left( \frac{1}{B_{44}} \frac{\partial^2}{\partial y^2} \right) \right] - 1 \right\} \Psi = 0 \end{aligned} \quad (3.10)$$

The system of equations (2.15)-(2.19) was broken down into two systems: (3.9) (plane problem) and (3.10) (bending without expansion-contraction of the average plane).

4. Cylindrical bending of a plate. Having directed axis  $x$  along the span of the plate, from (2.15)-(2.19), taking into account (2.7), we obtain

$$\begin{aligned} \frac{d}{dx} \left( C_{11} \frac{du}{dx} \right) - \frac{d}{dx} \left( K_{11} \frac{d^2 w}{dx^2} \right) + \frac{d}{dx} \left( C_{111} \frac{d}{dx} + C_{111}^x \frac{d^2}{dx^2} \right) \varphi = \frac{d(R_{110}^x - \theta_{10})}{dx} \\ d\varphi/dx = 0 \end{aligned} \quad (4.1)$$

$$\frac{d}{dx} \left( K_{11} \frac{du}{dx} \right) - \frac{d}{dx} \left( D_{11} \frac{d^2 w}{dx^2} \right) + \left[ \frac{d}{dx} \left( K_{111} \frac{d}{dx} + K_{111}^x \frac{d^2}{dx^2} \right) - \frac{h^3}{12} \right] \varphi = \frac{d(R_{111}^x - \theta_{11})}{dx}$$

From the second equation of (4.1) we find  $\varphi = \text{const}$ . Considering this, and having integrated once the first and third equations of (4.1), we will have

$$C_{11} \frac{du}{dx} - K_{111} \frac{d^2 w}{dx^2} = -C_{111}^x \varphi + R_{110}^x - \theta_{10} + C_1 \quad (4.2)$$

$$K_{11} \frac{du}{dx} - D_{11} \frac{d^2 w}{dx^2} = -K_{111}^x \varphi + \frac{h^3 \varphi}{12} x + R_{111}^x - \theta_{11} + C_2 \quad (4.3)$$

where  $C_1$  and  $C_2$  are constants of integration.

For definiteness, having taken  $C_{11} D_{11} - K_{11}^2 \neq 0$ , (4.2) and (4.3) we will obtain

$$\begin{aligned} \frac{du}{dx} = \frac{1}{C_{11} D_{11} - K_{11}^2} \left[ \varphi \left( K_{11} K_{111}^x - D_{11} C_{111}^x - K_{11} \frac{h^3 x}{12} \right) + \right. \\ \left. + C_1 D_{11} - C_2 K_{11} + D_{11} R_{110}^x - K_{11} R_{111}^x - D_{11} \theta_{10} + K_{11} \theta_{11} \right] \\ \frac{d^2 w}{dx^2} = \frac{1}{C_{11} D_{11} - K_{11}^2} \left[ \varphi \left( C_{11} K_{111}^x - K_{11} C_{111}^x - C_{11} \frac{h^3 x}{12} \right) + \right. \\ \left. + C_1 K_{11} - C_2 C_{11} + K_{11} R_{110}^x - C_{11} R_{111}^x - K_{11} \theta_{10} + C_{11} \theta_{11} \right] \end{aligned} \quad (4.4)$$

Integration (4.4) does not present any great difficulties for the known law of temperature change and for given functions which express the dependence of the deformation factor and the thermal coefficient of expansion on temperature.

The six constants of integration which enter into expressions  $\varphi$ ,  $u$  and  $w$  are determined from the conditions that the plate is attached

along the sides  $x = 0$  and  $x = 1$ . [There are three constants of integration  $\phi$ ,  $C_1$  and  $C_2$  in (4.4) and three also appear as a result of integration of (4.4).]

Example of the calculation. We will examine a case when zero temperature is maintained at the lower surface of the plate, while at the upper surface the temperature changes linearly from  $0^\circ$  to  $400^\circ$ , i.e.,

$$T = 400 \frac{z}{h} \text{ when } z = \frac{h}{2}, T = 0 \text{ when } z = -\frac{h}{2} \quad (4.5)$$

We can easily see that the temperature function

$$T = 400 \frac{z}{h} \left( \frac{z}{h} + \frac{1}{2} \right) \quad (4.6)$$

satisfies the known equation of heat transfer and the boundary conditions (4.5).

We will take [5]

$$\begin{aligned} \frac{E_z}{E} = \frac{G_{xy}}{G} = 1 - 0.0005T, \quad \beta_x = \beta_y = \beta = \text{const}, \\ \mu_{xy} = \mu_1 = \text{const}, \quad \mu_{yz} = \mu_2 = \text{const} \end{aligned} \quad (4.7)$$

Introducing (4.6) into (4.7) and introducing new variables

$$\eta = \frac{2z}{h}, \quad \omega = \frac{z}{10h} \quad (4.8)$$

from (2.9), (2.14) and (4.4) we will obtain the values of the first derivative of displacement  $\underline{u}$  and the second derivative of displacement  $\underline{w}$  with respect to variable  $\omega$ , which contain logarithmic terms.

Taking into account the boundedness of variable  $\omega$

$$0 \leq \omega \leq 1/10$$

and [6] under the condition  $1 \geq x \geq 0$ ] and given the accuracy of the calculations, we can substantially simplify the obtained results.

Next we expanded the logarithmic functions into power series and

having limited ourselves to an accuracy of  $1 + \omega^4 \approx 1$ , after integration we will obtain

$$u = u(\omega, \varphi, C_1, C_2, C_3), \quad w = w(\omega, \varphi, C_1, C_2, C_4, C_5) \quad (4.9)$$

Introducing (4.2) and (4.9) into (1.2), (2.10), (2.12), and (2.13), we can easily find the stresses, displacements, forces, and moments expressed by constants  $\varphi$  and  $C_1$  ( $i = 1, 2, \dots, 5$ ), the values of which are determined from the conditions that the plate is attached at the edges  $x = 0$  and  $x = l$  (correspondingly,  $\omega = 0$  and  $\omega = 0.1$ ).

We will examine a case when along the sides  $x = 0$  and  $x = l$  where  $\eta = z = 0$ , the following conditions are satisfied.

$$u_x - u_z = \frac{\partial u_z}{\partial x} = 0 \quad (4.10)$$

From (2.10), (4.9), and (4.10), taking into account the boundedness  $\eta(|\eta| \leq 1)$ , we will have

$$\begin{aligned} C_2 = C_4 = C_5 = 0, \quad \varphi = -\frac{337.4}{1 - 0.001163\lambda^2} \cdot \frac{E^*}{M} \\ C_1 = -\frac{92.59 + 0.1481\lambda^2}{1 - 0.001163\lambda^2} E^* \lambda, \quad C_3 = -\frac{0.6365 + 0.2798\lambda^2}{1 - 0.001163\lambda^2} E^* \lambda^2 \end{aligned} \quad (4.11)$$

Here

$$\lambda^2 = \frac{E}{G(1 - \mu_1 \mu_2)} \frac{\lambda^3}{h^3}, \quad E^* = E \frac{B(1 + \mu_2)}{1 - \mu_1 \mu_2}$$

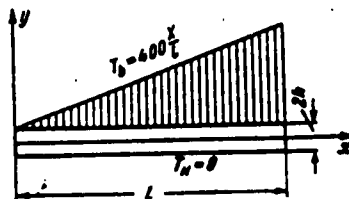
Obviously from (4.11), in the case  $h^* = 0.114 - 1.00$ , the correction resulting from taking into account transverse shear when determining  $C_2$  can reach 5-44%.

Introducing (4.4) into (2.13), for the deflecting moment  $M_x$  we will have

$$M_x = 0.0833 \varphi \lambda^2 x + C_1$$

from which it is evident that taking into account the influence of transverse shear can in certain cases very substantially influence the calculated values (for example, the value of the deflecting moment  $M_x$  in cross section  $x = 0$  in the mentioned example).

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#### REFERENCES

1. S. A. Ambartsumyan. Teoriya anizotropnykh obolochek. Fizmatgiz, M., 1961.
2. L. S. Leybenzon. Kurs teorii uprugosti. Gostekhizdat., M.-L., 1947.
3. S. A. Ambartsumyan. Temperaturnyye napryazheniya v sloistnykh anizotropnykh obolochkakh. Izv. AN Arm. SSR, Vol. V, No. 6, 1952.
4. S. A. Ambartsumyan. K teorii izgiba anizotropnykh plastinok. Izv. AN SSSR, OTH, No. 5, 1958.
5. S. M. Durgar'yan. Temperaturnyy raschet ortotropnoy sloistoy plastinki pri uprugikh postoyannykh i koeffitsiyente temperaturnogo rasshireniya, zavisyashchikh ot temperatury. Izv. AN Arm. SSR, Seriya fiz.-mat. Nauk, 13, No.2, 1960.

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